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Resulting Errors of Measurement Chains

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Abstract

This paper presents a theoretical analysis of error calculations during a system design. A system is understood as a measurement chain that consists of units called transducers. In general, transducers are connected in three basic configurations; serial, parallel and with feedback. In order to illustrate the analysis of error calculations we consider only two connected units for each configuration of a measurement chain. In this paper, we analyze two types of error calculations for these three basic configurations. First, a standard error calculation is described assuming that transducer errors in a measurement chain are mutually correlated. System designers frequently assume an error correlation and therefore use standard error calculations. Second, error calculations are performed assuming that transducer errors are not mutually correlated. The case of mutually uncorrelated transducer errors is very common in a real system design since the numerical specifications of individual transducers are obtained from multiple independent catalogs. Thus, it is the uncorrelated nature of transducer errors that requires a modification of standard error calculations. We analyze error calculations for each configuration of transducers and for mutually correlated and uncorrelated transducer errors. In conclusion, the error formulas assuming mutually uncorrelated transducer errors model a real system design more accurately than the standard error formulas.

1. Introduction

It is well known that a typical measurement chain consists of a chain of measuring units called transducers. The first unit of a measurement chain is an appropriate sensor that converts physical phenomena into a quantitative entity (signal) suitable for further processing. Although a functionality of transducers in engineering applications varies significantly, the configuration of transducers in a measurement chain is limited by three basic schemes for connecting transducer units. The three configurations are serial, parallel and with feedback as they are shown in Figures 1, 2 and 3. In this paper, we will present two types of error calculations for these three basic configurations performed by a system designer. The types of error calculations will depend on a particular configuration and a mutual correlation of unit errors. For simplicity, we will consider that each configuration of a measurement chain contains only two connected units and numerical specifications of individual units will be obtained from unit catalogs.

2. Preliminaries

Each of the three basic configurations is described by a linear transfer function between an input signal x and an output signal y. The transfer functions are expressed in Equation (1a) for a serial configuration (Fig. 1), Equation (1b) for a parallel configuration (Fig. 2) and Equation (1c) for a configuration with feedback (Fig. 3).

Serial configuration:

$$y = k_1 k_2 x \tag{1a}$$

where k_1 and k_2 are transfer constants of transducers T_1 and T_2.

Parallel configuration:

$$y = (k_1 \pm k_2)\, x \tag{1b}$$

where the plus sign is applied if signals from transducers T_1 and T_2 are added and the minus sign is used if two signals are subtracted.

Configuration with feedback:

$$y = \frac{k_1}{1 \mp k_1 k_2} x \tag{1c}$$

where the minus sign is used for a positive feedback and the plus sign is for a negative feedback.

In general, stability of k_1 and k_2 parameters depends on internal and external conditions of a measurement chain, for example, temperature and humidity of an environment or amplitude and frequency of supplied voltage. A manufacturer specifies the impact of all conditions on accuracy of a transducer as various errors (temperature, hysteresis, frequency error, etc.). The error values are commonly given as relative values.

Let us assume that the relative errors of two transducers T_1 and T_2 are known and are denoted as dk_1 and dk_2. According to the Law of error propagation [1], [2] and [3], the resulting relative error dy of an output signal y is characterized by the following equation

$$dy = \pm(A + B) \tag{2}$$

where

$$A = \frac{\partial y}{\partial k_1} \frac{k_1}{y} dk_1$$

and

$$B = \frac{\partial y}{\partial k_2} \frac{k_2}{y} dk_2$$

The presence of +/- signs in Equation (2) denotes the upper and lower bounds of resulting error δy.
The A and B components are defined in Equations (2a), (2b) and (2c) depending on a configuration of a measurement chain.

Serial configuration:

$$A = \pm dk_1$$
$$B = \pm dk_2 \tag{2a}$$

Parallel configuration:

$$A = \pm \frac{k_1 dk_1}{k_1 \pm k_2}$$

$$B = \pm \frac{k_2 dk_2}{k_1 \pm k_2} \tag{2b}$$

Configuration with feedback:

$$A = \pm \frac{dk_1}{1 \mp k_1 k_2}$$

$$B = \pm \frac{k_1 k_2}{1 \mp k_1 k_2} dk_2 \tag{2c}$$

The signs in the denominators of A and B correspond to the type of signal combination, e.g., signal addition (upper sign) or signal subtraction (lower sign) in Equations (2b). The error dy in Equation (2) using A and B from Equations (2b) will be minimized if the plus signs in denominator are applied.

Similarly, the signs in the denominators of A and B in Equation (2c) correspond to configurations with positive feedback (upper sign) or negative feedback (lower sign). It is also apparent that the error dy calculated from Equations (2) and (2c) will be smaller for a configuration with negative feedback (plus signs in the denominators) than for a configuration with positive feedback.

It is known that the negative feedback is used in measurement chains and automated control systems. In this case the signs A and B are opposite and it is possible to achieve the resulting error to be zero ($dy = 0$) if Equation (3) is satisfied.

$$dk_1 = k_1 k_2 dk_2 \tag{3}$$

On the other hand, the positive feedback is currently used in radio electronics during signal generation. The resulting error in this case is an addition of two positive or two negative terms expressed in Equation (2c).

3. Results

The aforementioned Equations (2), (2a), (2b) and (2c) are defined in [1], [2] and [3] and follow the Law of error propagation. The formulas above assume that the transducer errors are mutually correlated. It means that the correlation coefficient r is assumed to be $r = \pm 1$.

However, the assumption about mutually correlated errors of transducers is not satisfied in reality. The error values of transducers are obtained by a system designer from multiple manufacturer's catalogs, therefore the individual errors are uncorrelated and the mutual correlation coefficient is zero ($r = 0$). In this case we must calculate the resulting relative error δy as the square root of a sum of squared error components. This is based on the calculation of uncorrelated uncertainties [4], [5]. The mathematical formulation is shown in Equation (4) and Equations (4a) and (4b) respectively.

$$dy = \pm\sqrt{A^2 + B^2} \qquad (4)$$

Let us compare Equations (2) and (4) in the case of serial or parallel configurations. We can see that the resulting relative error calculated for correlated unit errors is larger that the resulting error for uncorrelated unit errors. For instance, let us assume that two transducers T_1 and T_2 have identical uncorrelated relative errors ($dk_1 = dk_2 = dk$) and are in a serial configuration. The resulting error is equal

$$dy = \sqrt{2}dk = 1.41dk \qquad (4a)$$

as opposed to the case of correlated transducer errors when the resulting error is equal $dy = 2\ dk$.

If two transducers are in a parallel configuration and have identical uncorrelated errors, then the resulting error is equal to

$$dy = 0.5\sqrt{2}dk = 0.7dk \qquad (4b)$$

This resulting error is smaller than the error of one transducer.

Let us perform the same comparison of Equations (2) and (4) for the configuration with negative feedback. This comparison leads to a conclusion that the resulting error does not change significantly regardless of a mutual correlation of transducer errors. One can observe that the resulting error dy for uncorrelated transducer errors (Equation (4) with A and B from Equation (2c)) is larger than the error dy for correlated transducer errors (Equation (2) with A and B from Equation (2c)).

In summary, we have shown two mathematical formulations for an error computation of measurement chains. The two formulations are expressed in Equations (2) or (4) and should be used appropriately with the data obtained from measurements (Equation (2)) or manufacturer's catalogs (Equation (4)). It is also important to notice that the coefficients k_1 and k_2 play role only in Equations (2b) and (2c). Equation (2a) is independent of the coefficients k_1 and k_2 and the resulting error is composed of relative errors only.

4. Conclusion

In this paper, we demonstrated that system designers should calculate the resulting relative error of measurement chains according to not only a configuration of measurement chain but also a mutual correlation of transducer errors. It is the uncorrelated nature of transducer errors obtained from multiple catalogs of manufacturers that requires a modification of the standard error calculations performed by system designers. The presented error formulas model a real design of measurement chains more accurately than the standard error formulas.

References

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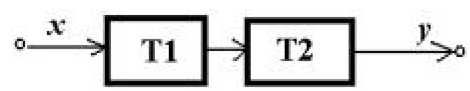

Figure 1: Serial configuration.

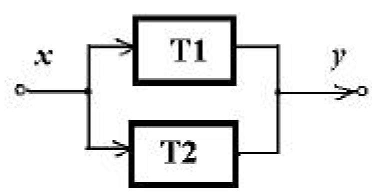

Figure 2: Parallel configuration.

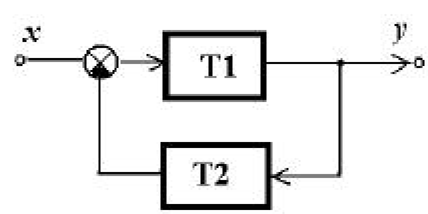

Figure 3: Configuration with feedback.